Calculus Warm Up #1-5

For each of the following derivatives, f '(x), imagine working backwards. What could be the equation of f(x)?

1) $f'(x) = 2x$

2) $f'(x) = 3x^2$

Is that the only possibility for f(x)?

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33) $r(t) = t^2 i + t^3 j$

$r'(t) = 2t i + 3t^2 j$

$r'(0) = 0i + 0j$

Smooth $\rightarrow$ $(-\infty, 0) \cup (0, \infty)$

35) $r(\theta) = \langle 2(\cos\theta)^3, 3(\sin\theta)^3 \rangle$

$r'(\theta) = \langle -6\cos^2\theta \sin\theta, 9\sin^2\theta\cos\theta \rangle$

$r'(\theta) = 0$ when $\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

$\theta \neq \frac{\pi}{2} n$

$\boxed{\text{Smooth on } (-\infty, \infty) \setminus \{\frac{\pi}{2} n\}}$

$(\frac{\pi}{2} n, \frac{\pi}{2}(n+1))$

37) $r'(\theta) = 0$ when

$\theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{\pi}{3} + \pi n$

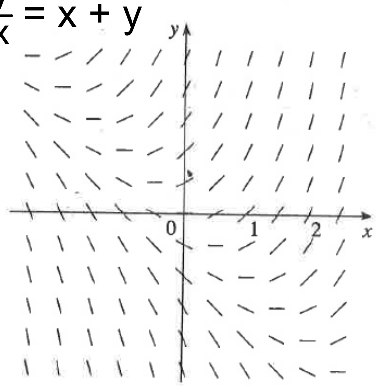
30) $r(t) = \langle \cot t\ ,\ \sin 2t \rangle$

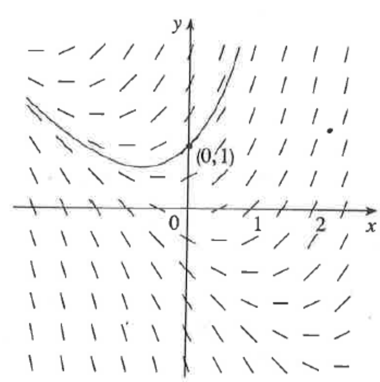
$r'(t) = \langle -\csc^2 t\ ,\ 2\cos 2t \rangle$

The slope field represents the family of possible curves given the derivative.

Given the initial condition that the curve passes through (0,1), we can find a particular solution.

$\frac{dy}{dx} = x + y$





5.1 Day 1

-Antiderivatives

-Notation for Antiderivatives

-Basic Integration Rules

$$F(x) = x^3 \quad \text{because} \quad \frac{d}{dx}[x^3] = 3x^2.$$

F is an antiderivative of $f(x) = 3x^2$

f(x) could have been... $3x^2 + 5$ or $3x^2 - 8$ or ...

Theorem 5.1

If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form

$$G(x) = F(x) + C, \quad \text{for all } x \text{ in } I$$

where C is a constant.

G represents the entire family of antiderivatives of *f*, where C is an arbitrary constant.

G is called the general antiderivative of *f*

Notation for antiderivatives:

The indefinite integral   Where $F'(x) = f(x)$

Integral Symbol

Variable of integration

$$y = \int f(x)\, dx = F(x) + C.$$

Integrand

Constant of integration

The antiderivative of f with respect to x.

The inverse nature of integration and differentiation can be verified by substituting $F'(x)$ for $f(x)$ in this definition to obtain

$$\int F'(x)\, dx = F(x) + C.$$

Integration is the inverse of differentiation

Moreover, if $\int f(x)\, dx = F(x) + C$, then

$$\frac{d}{dx}\left[\int f(x)\, dx\right] = f(x).$$

Differentiation is the inverse of integration

This allows us to obtain integration formulas directly from differentiation formulas, as shown in the following theorem.

Theorem 5.2, p. 232 Basic Integration Rules

Differentiation formula	*Integration formula*
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx] = k,\ k \neq 0$	$\int k\,dx = kx + C,\ k \neq 0$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\,dx = k\int f(x)\,dx$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

Let's try it! Undoing the power rule:

$$\int 3x\,dx = \frac{3x^2}{2} + C$$

Evaluate $\int (3x^2 + 2x)\,dx$.

$$= \frac{3x^3}{3} + \frac{2x^2}{2} + C$$

$$F(x) = x^3 + x^2 + C$$

Rewriting before integrating

Given integral	*Rewrite*
(a) $\int \frac{1}{x^3}\,dx$	$\int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$
	$= -\frac{1}{2x^2} + C$
(b) $\int \sqrt{x}\,dx$	$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2}$
	$= \frac{2x^{3/2}}{3} + C$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int\left(\frac{x^1}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx$$

$$= \int (x^{1/2} + x^{-1/2})\,dx$$

$$= \frac{2x^{3/2}}{3} + \frac{2x^{1/2}}{1} + C$$

$$\int \frac{3x^2-4}{x^2}\,dx. = \int (3 - 4x^{-2})\,dx$$

$$= 3x - \frac{4x^{-1}}{-1} + C$$

$$= 3x + \frac{4}{x} + C$$

HW:

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